COMMUNICATION THEORY

GATE / PSUs

ELECTRONICS ENGINEERING-ECE

STUDY MATERIAL

COMMUNICATION THEORY

ENGINEERS INSTITUTE OF INDIA

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CONTENT

CHAPTER-1
RANDOM SIGNALS & NOISE

1. Random Signals:

Signals can be classified as deterministic and random. Deterministic signals can be completely specified as functions of time. Random signals are those whose value cannot be predicted in advance. Radio communication signals are random signals, which are invariably accompanied by noise. Noise like signals, also called random processes can be predicted about their future performance with certain probability of being correct.

Probability:

The concept of probability occurs naturally when we contemplate the possible outcomes of an experiment whose outcomes are not always the same. Thus the probability of occurrence of an event A is

$$P(A) = \frac{\text{number of possible favourable outcomes}}{\text{total number of possible equally likely outcomes}} \quad \text{... (i)}$$

Two possible outcomes of an experiment are defined as being mutually exclusive if the occurrence of one outcome precludes the occurrence of the other. Hence,

$$P(A_1 \text{ or } A_2) = P(A_1) + (A_2) \quad \& \; P(A \cap B) = 0 \quad \text{... (ii)}$$

Where A_1 and A_2 are two mutually exclusive events.

The joint probability of related events is given as:

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \quad \text{... (iii)}$$

Where $P\left(\frac{B}{A}\right)$ is the conditional probability of outcome B, given A has occurred and P(A, B) is the probability of joint occurrence of A and B.

2. Random Variables:

The term random variable is used to signify a rule by which a real number is assigned to each possible outcome of an experiment.

A random experiment with sample space S. A random variable $X(\lambda)$is a single valued real function that assigns a real number to each sample point.

The sample space S is termed as domain of random variable X and the collection of all numbers [values of] is termed as the range of random variable X.

$$P(X = x) = P\{X(\lambda) = x\}$$
$$P(X \le x) = P\{X(\lambda) \le x\}$$
$$P(x_1 < x \le x_2) = P\{\lambda : x_1 < X(\lambda) \le x_2\}$$

If X can take on only a countable number of distinct values, then X is called a discrete random variable.

If X can take any values within one or more intervals on the real line, then X is called a continuous random variable.

2.1 Probability Mass function (PMF):

$P_X(x) = P(X = x)$

It specifies that what is the probability that random variable X taking each of its possible value ***x***.

2.2 Cumulative Distribution Function (CDF):

The cumulative distribution function associated with a random variable is defined as the probability that the outcome of an experiment will be one of the outcome for which X(A) ≤ x, where x is any given number. If $F_X(x)$ is the cumulative distribution function then

$$F_X(x) = P(X \le x)$$

where P (X ≤ x) is the probability of the outcome.

$F_X(x)$ has the following properties:

- $0 \le F(x) \le 1$
- $F(-\infty) = 0,\ F(\infty) = 1$
- $F(x_1) \le F(x_2)$ if $x_1 < x_2$.

2.3 Probability Density Function (PDF):

The probability density function $f_X(x)$ is defined in terms of cumulative distribution function $F_X(x)$ as:

$$f_X(x) = \frac{d}{dx} F_x(x)$$

The properties of PDF are:

- $f_X(x) \ge 0$, for all x
- $\int_{-\infty}^{\infty} f_x(x)\,dx = 1$
- $F_x(x) = \int_{-\infty}^{x} f_x(x)\,dx$

The relation between probability and probability density can be written as:

$$P(x_1 \le x \le x_2) = \int_{x_1}^{x_2} f(x)\,dx$$

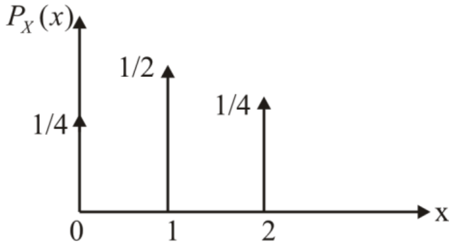
Example : A random variable X specifies the number of possible heads in the experiment of tossing a coin twice construct the PMF, PDF & CDF.

Solution: Sample space $S = \{HH, TT, TH, HT\}$

Now $P_X(0) = P(X = 0) = \frac{1}{4}$

$$P_X(1) = P(X = 1) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

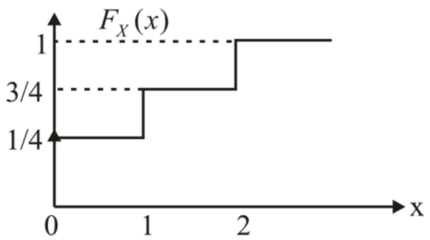
$$P_X(2) = P(X = 2) = \frac{1}{2}$$



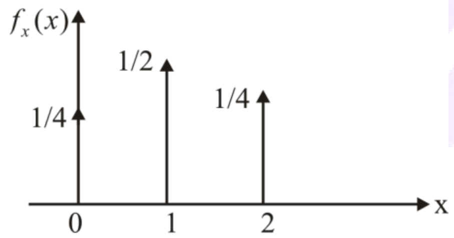
$$CDF: \quad F_X(0) = P(X \le 0) = \frac{1}{4}$$

$$F_X(1) = P(X \le 1) = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$

$$F_X(2) = P(X \le 2) = \frac{1}{4} + \frac{1}{2} + \frac{1}{2} = 1$$



$$PDF: \quad f_X(x) = \frac{d}{dx} F_X(x)$$



Example: Consider the probability f (x) = $ae^{-b|x|}$, where X is a random variable whose allowable values range from x = –∞ to x = +∞.

Find:

a. The cumulative distribution function F (x).

b. The relationship between a and b

c. The probability that the outcome x lies between 1 and 2.

Solution:

a) The cumulative distribution function is :

$$F(x) = P(X \le x)$$

$$= \int_{-\infty}^{x} f_x(x)\,dx$$

$$= \int_{-\infty}^{x} ae^{-b|x|}dx$$

$$= \begin{cases} \frac{a}{b}e^{bx}, & x \le 0 \\ \frac{a}{b}(2 - e^{-bx}) & x \ge 0 \end{cases}$$

$$\therefore \quad \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\therefore \quad \int_{-\infty}^{\infty} ae^{-b|x|}dx = \frac{2a}{b} = 1$$

$$\Rightarrow \quad \frac{a}{b} = \frac{1}{2}$$

c) The probability that x lies in the range between 1 and 2 is

$$P(1 \le x \le 2) = \frac{b}{2}\int_{1}^{2} e^{-b|x|}dx = \frac{1}{2}(e^{-b} - e^{-2b})$$

2.4 Statistical averages of random variable:

Mean :The average value of a random variable is given as:

$\bar{X} = E(X) = m = \sum_i x_i P(x_i)$,Where $\bar{X}$ or E(x) or m represents the average, expected or mean value of random variable X.

Variance : The variance (σ^2) x is given as

$$\sigma^2 = E(x^2) - m^2$$

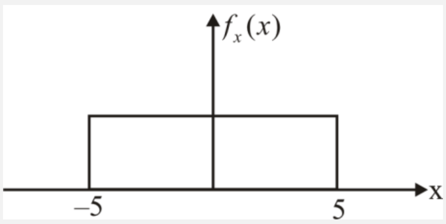
Where σ is the standard deviation of X, & $E(x^2)$ is mean square value.

If the average value *m* = 0 then,

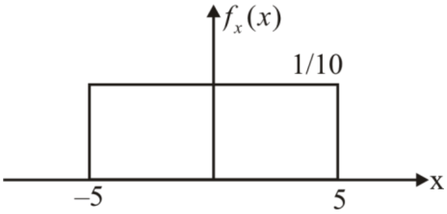
$$\sigma^2 = E(X^2)$$

Note : When random variables are independent, they are uncorrelated but vice versa is not true.

Example: For A continuous random variable X Pdf is given below, find mean, mean square value & standard deviation



Solution:



$$Mean = E[X] = \int_{-5}^{5} x.\frac{1}{10}dx = \frac{1}{10}\left(\frac{x^2}{2}\right)_{-5}^{5} = 0$$

$$MSV = E[X^2] = \int_{-5}^{5} x^2 \frac{1}{10}dx = \frac{1}{10}\left(\frac{x^3}{3}\right)_{-5}^{5} = 8.33$$

$$S.O. = \sqrt{E[X^2] - (E(X))^2} = 2.88$$

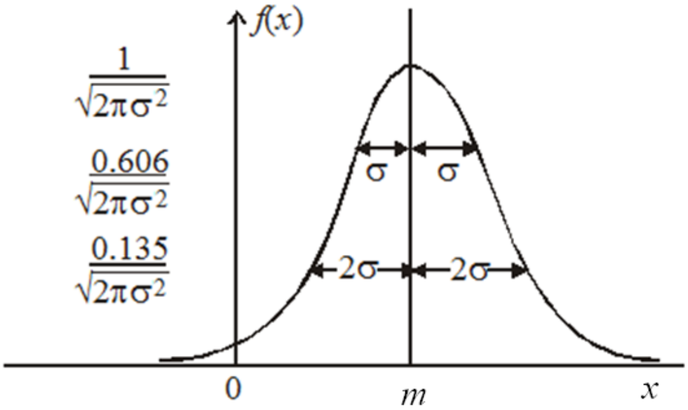
2.5 Standard PDF:

Gaussian PDF:

The Gaussian PDF is of great importance in the analysis of random variables. It is defined as

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-m)^2}{2\sigma^2}}$$

And is plotted in figure below, m and σ^2 are the average value and variance of f(x).

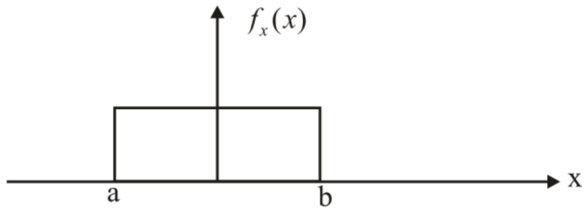


Thus, $x = m = \int_{-\infty}^{\infty} x f(x)\, dx$ and

$$E[(x-m)^2] = \int_{-\infty}^{\infty} (x-m)^2\, f(x)\, dx = \sigma^2$$

It can be deduced from figure that when x - m = ± σ, f (x) has fallen to 0.606 of its peak value. When x = m ± 2σ, f (x) falls to 0.135 of the peak value.

Uniform density function:



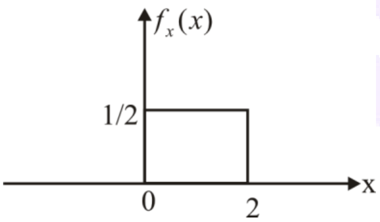
$$f_X(x) = \frac{1}{b-a},\ a \le x \le b$$
$$= 0,\ otherwise$$

2.6 Differential entropy: It is defined for continuous random variables.

$$H(X) = \int_{-\infty}^{\infty} f_X(x) \log_2\left(\frac{1}{f_X(x)}\right) dx$$

Example: A continuous random variable X is uniformly distributed in (0, 2). Find differential entropy of X.

Solution:



$$H(X) = \int_{-\infty}^{\infty} f_X(x) \log_2\left(\frac{1}{f_X(x)}\right) dx$$
$$= \int_0^2 \frac{1}{2} \log_2 2dx = \frac{1}{2} \times 2 = 1 \frac{bit}{symobol}$$

2.7 Joint mass function:

$$P_{XY}(x,\ y) = P(X = x,\ Y = y)$$

$$\Rightarrow \sum_i \sum_j P_{XY}(X_i,\ Y_j) = 1$$

$$\&\ \ P(x_1) = \sum_{j=1}^{n} P_{XY}(x_1,\ y_j)$$

$$\&\ \ P(y_1) = \sum_{i=1}^{m} P_{XY}(x_i,\ y_1)$$

2.8 Joint density function : $(f_{XY}(x, y))$

(i) $$\int_{-\infty}^{\infty}\int_{\infty}^{\infty} f_{XY}(x, y)\,dx\,dy = 1$$

(ii)

$$\left.\begin{aligned} f_X(x) &= \int_{-\infty}^{\infty} f_{XY}(x, y)\,dy \\ f_Y(y) &= \int_{-\infty}^{\infty} f_{XY}(x, y)\,dx \end{aligned}\right]\text{marginal density functions}$$

2.9 Joint distribution function:

$$F_{XY}(x, y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{XY}(x, y)\,dx\,dy.$$

Example : Joint density function is given by $f_{XY}(x, y) = e^{-(x+y)},\ x \ge 0,\ y \ge 0 = 0\ otherwise$

Find marginal density functions.

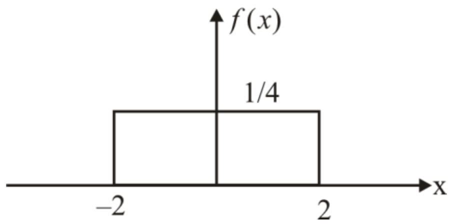
Solution: $f_X(x) = \int_{-\infty}^{\infty} e^{-(x+y)}dy = e^{-x}\left[\frac{e^{-y}}{-1}\right]_0^{\infty} = e^{-x},\ x \ge 0$

Now, $f_Y(y) = \int_{-\infty}^{\infty} e^{-(x+y)}dx \ = e^{-y},\ y \ge 0$

Example: A continuous random variable X is uniformly distributed in [–2, 2] find

$f(y)$ such that $y = \sin x$

Solution:



$$Now \quad f(y) = f(x)\left|\frac{dx}{dy}\right|$$
$$= \frac{1}{4}\left|\frac{d}{dy}\sin^{-1} y\right|$$
$$= \frac{1}{4}\frac{1}{\sqrt{1-y^2}}$$

3. **Random Process**:

- Let **Ψ** denote the random outcome of an experiment. To every such outcome suppose a waveform X(t, **Ψ**) is assigned. The collection of such waveforms form a random process.

X(t, **Ψ**) is a specific time function.

- For fixed t, $X_1 = X(t_1, \Psi_1)$ is a random variable. The ensemble of all such realizations X(t, **Ψ**) over time represents the stochastic process X(t).

For example $X(t) = A\cos(w_0 t + \theta)$ where θ is a uniformly distributed random variable in (0,2π) represents a random process.

- If X(t) is a random process, then for fixed t, X(t) represents a random variable. Its distribution function is given by
$$F_X(x,t) = P\{X(t) \le x\}$$

- Here $F_X(x,t)$ depends on t, since for a different t, we obtain a different random variable.
Now density function is given by
$$f_X(x,t) \triangleq \frac{dF_X(x,t)}{dx}$$
For $t = t_1$ and $t = t_2$, $X(t)$ represents two different random variables $X_1 = X(t_1)$ *and* $X_2 = X(t_2)$ respectively. Their joint distribution is given by
$$F_X(x_1, x_2, t_1, t_2) = P\{X(t_1) \le x_1, X(t_2) \le x_2\}$$

- And the joint density function (2nd order density function) is given by
$$f_X(x_1, x_2, t_1, t_2) \triangleq \frac{\partial^2 F_X(x_1, x_2, t_1, t_2)}{\partial x_1\, \partial x_2}$$

3.1 Statical Averages of Random Process:

Mean of a Random Process:

$$E\{X(t)\} = \int_{-\infty}^{+\infty} x f_X(x,t)\, dx$$

Represents the mean value of a process X(t).

Autocorrelation:

$$R_{XX}(t_1,t_2) \triangleq E\{X(t_1)X^*(t_2)\} = \iint x_1 x_2^* f_X(x_1, x_2, t_1, t_2)\, dx_1\, dx_2$$

Properties of Autocorrelation:

1. $R_{XX}(t_1,t_2) = R_{XX}^*(t_2,t_1) = [E\{X(t_2)X^*(t_1)\}]^*$
2. $R_{XX}(t,t) = [E\{|X(t)|^2\} > 0$ (Average instantaneous power)
3. $R_{XX}(t_1,t_2)$ represents a nonnegative definite function, i.e., for any set of constants $\{a_i\}_{i=1}^{n}$
$$\sum_{i=1}^{n}\sum_{j=1}^{n} a_i a_j^* R_{XX}(t_i, t_j) \ge 0$$

The **auto covariance** function of the process X(t) is:

$$C_{XX}(t_1,t_2) = R_{XX}(t_1,t_2) - \mu_X(t_1)\mu_X^*(t_2)$$

Example: If X(t) = A cos (ω_0t + θ), Determine the mean and autocorrelation function of X(t) where θ is a uniformly distributed random variable in (0,2π).

$$\mu_X(t) = E\{X(t)\} = aE\{\cos(\omega_0 t + \varphi)\}$$
$$= a\cos\omega_0 tE\{\cos\varphi\} - a\sin\omega_0 tE\{\sin\varphi\} = 0$$
$$\text{since } E\{\cos\varphi\} = \frac{1}{2\pi}\int_0^{2\pi}\cos\varphi\, d\varphi = 0 = E\{\sin\varphi)$$

Now,

$$R_{XX}(t_1,t_2) = a^2E\{\cos(\omega_0 t_1 + \varphi)\cos(\omega_0 t_2 + \varphi)$$
$$= \frac{a^2}{2}E\{\cos\omega_0(t_1 - t_2) + \cos(\omega_0(t_1 + t_2) + 2\varphi)\}$$
$$= \frac{a^2}{2}\cos\omega_0(t_1 - t_2)$$

3.2 Time averages:

(i) Mean

$$< X(t) >= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} X(t)dt$$

(ii) Mean square value:

$$< X^2(t) >= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} X^2(t)dt$$

(iii) Variance $= MSV - (mean)^2$

Note: If ensemble averages is same as time average then it is called ergodic random process,

3.3 Stationary Random Processes: Stationary processes exhibit statistical properties that are invariant to shift in the time index.
For example: {X(t_1) , X(t_2) } = {X(t_1+c) , X(t_2+c)} for any c.

A process is nth-order **Strict-Sense Stationary (S.S.S)** if

$$f_X(x_1,x_2,....,x_n,t_1,t_2...,t_n) \equiv f_X(x_1,x_2,....x_n,t_1 + c...,t_n + c)$$

For a **first-order strict sense stationary process**, we have

$$f_X(x,t) = f_X(x,t + c)$$

for any c. If we put c= –t then

$$f_X(x,t) = f_X(x) \qquad ...(1)$$

i.e., the first-order density of X(t) is independent of t. So

$$E[X(t)] = \int_{-\infty}^{+\infty} xf(x)\,dx = \text{constant}$$

Similarly, for a **second-order strict-sense stationary process**

$$f_X(x_1,x_2,t_1,t_2) \equiv f_X(x_1,x_2,t_1 + c,t_2 + c)$$

CHAPTER-2
ANALOG MODULATION SCHEMES

Communication is a process of exchanging information between one terminal and other terminal or between transmitter and receiver. There are two types of communication system.

A. Wired communication

B. Wireless communication

A. Wired communication:- in this type of communication channel used between transmitter and receiver is wired channel. This type of communication is used for small distance.

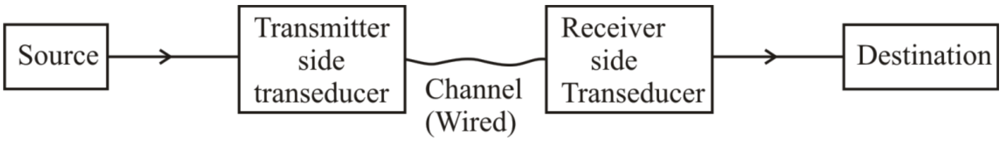


Figure: Block diagram of wired communication.

Note: Transducer is used to covert physical signal to electrical signal (Transmitter side) or electrical signal to physical signal. (Receiver side)

B. Wireless communication:- For long distance wireless communication is used with modulation.

Modulation: Modulation process performs systematic alternation of one waveform with the characteristic of another waveform.

Need of modulation:-

1. To reduce the height of antenna:-

Height of antenna = $\frac{\lambda}{4}$ …(i)

Where, λ is wavelength of the signal

Now, $\lambda = \frac{C}{f}$

$\Rightarrow \lambda = \frac{3\times10^8}{f}$

$\Rightarrow \lambda \propto \frac{1}{f}$ …(ii)

Where, f is signal frequency, from (i) and (ii) it is clear that height of antenna $\propto \frac{1}{f}$, so modulation reduces the height of antenna.

2. Ease of radiation
3. Efficient transmission

In this, modulation results in frequency translation. Base band message spectrum translated to pass band of the channel.

Note: Base band signals are those which are produced directly from the source without any modulation. They have significant content around DC.

Eg: Speech signal 300Hz – 3400Hz

Video signal 5MHz.

4. Leads to multiplexing

   It is possible to transmit different message signals into the same pass band of the channel.
5. Improves signal to noise ratio.

   Frequency modulation and phase modulation improves signal to noise ratio at the receiver output though transmission, bandwidth requirement is high.
6. It is used for frequency assignment for different stations.

Important property for modulation:

(1) Frequency shifting property

If $m(t) \xleftrightarrow{F.T.} M(f)$

then $m(t)e^{J2\pi f_0 t} \xleftrightarrow{F.T.} M(f-f_0)$

and $m(t)e^{-J2\pi f_0 t} \xleftrightarrow{F.T.} M(f+f_0)$

$\Rightarrow$ $A\cos(2\pi f_0 t) \xleftrightarrow{F.T.} \frac{A}{2}\left[\delta(f-f_0)+\delta(f+f_0)\right]$

(2) Modulation property:

$$m(t)\cos 2\pi f_0 t \xleftrightarrow{F.T.} \frac{M(f-f_0)+M(f+f_0)}{2}$$

Analog Communication:

In this type of communication information is transmitted by modulating a continuous signal i.e., in analog communication the message signal is an analog signal.

Mainly there are 3 types of modulation schemes in analog commutation, amplitude modulation, frequency modulation and phase modulation.

Type of modulation:-

i. **Single tone modulation**:- In this type of modulation message signal contains signal frequency component. For example, m(t) = $A_m \cos \omega_m t$

ii. **Multi tone modulation**:- In this type of modulation message signal contains multiple frequency component. For example, $m(t) = A_{m_1}\cos\omega_{m_1} t + A_{m_2}\cos\omega_{m_2} t$

Concept of modulation:

Message signal: The signal user want to send to the receiver. This signal also known as information signal or baseband signal or low frequency signal or modulating signal.

Carrier signal:- The signal which is used to carry the message signal. This is high frequency signal and generally denoted as C(t)

Now modulation is the process in which one of the characteristics parameter (Amplitude, phase or frequency) of the carrier signal will be varied linearly with respect to message signal amplitude variation.

Amplitude modulation:- It is the process in which amplitude of the carrier signal varies with message signal amplitude variation.

Analog Modulation Schemes:

Solution:

→ $f_{m1} = 1500\ Hz,\ A_{m1} = 3.8V\ \&\ \Delta f_1 = 9.8kHz$

⇒ $\Delta f_1 = k_f A_{m1}$

⇒ $9.8 = k_f 3.8 \quad \Rightarrow \quad k_f = 2.57 \dfrac{kHz}{V}$

Now $\beta_1 = \dfrac{\Delta f_1}{f_{m1}} = \dfrac{9.8}{1.5} = 6.533$

→ $f_{m2} = 1500\ Hz,\ A_{m2} = 9V,\ \Delta f_2 = k_f A_{m2} = 2.57 \times 9 = 23.13kHz$

$\&\ \beta_2 = \dfrac{23.13}{1.5} = 15.42$

→ $f_{m3} = 1200\ Hz,\ A_{m3} = 11V,\ \Delta f_3 = k_f A_{m3} = 28.27kHz$

$\&\ \beta_3 = \dfrac{28.27}{1.2} = 23.55$

Power calculation:

$$P_t = P_c + P_{f_c+f_m} + P_{f_c-f_m} + P_{f_c+2f_m} + P_{f_c-2f_m} + ------$$

$$P_t = \frac{A_c^2 J_0^2(\beta)}{2R} + \frac{A_c^2 J_1^2(\beta)}{2R} + \frac{A_c^2 J_1^2(\beta)}{2R} + \frac{A_c^2 J_2^2(\beta)}{2R} + \frac{A_c^2 J_2^2(\beta)}{2R} + -----$$

$P_t = \dfrac{A_c^2}{2R}$ [From the 3rd property of Bessel function]

Note: Power in 1st order S.B = $\dfrac{A_c^2 J_1^2(\beta)}{R}$

Example: The time domain expression of FM is $s(t) = 20\cos\left[2\pi \times 10^6 t + 8\sin 4\pi \times 10^3 t\right]$. Calculate β, Δf bandwidth and power.

Solution: $A_c = 20,\ \beta = 8,\ f_m = 2kHz$

⇒ $\dfrac{\Delta f}{f_m} = 8$

⇒ $\Delta f = 16\ \text{kHz}$

⇒ $P_t = \dfrac{A_c^2}{2} = 200\ \text{watt}$

⇒ $BW = 2(8+1) \times 2 = 36\ \text{kHz}$

Example : For FM signal given below, determine the modulation index, bandwidth and total power, after modulation

$$s(t) = 100\cos(2\pi \times 10^6 t + 3.8\sin 2\pi \times 10^3 t)$$

Solution:

$A_c = 100,\ f_c = 1MHz,\ f_m = 1kHz\ \&\ \beta = 3.8$

Here $\beta > 1$ ⇒ wideband modulation